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Taking as starting point a Lorentz and CPT non-invariant Chern-Simons-like model defined in 1+3 dimensions, we proceed realizing its dimensional reduction to $D = 1 + 2$. One then obtains a new planar model, composed by the Maxwell-Chern-Simons (MCS) sector, a Klein-Gordon massless scalar field, and a coupling term that mixes the gauge field to the external vector, v^{μ}. In spite of breaking Lorentz invariance in the particle frame, this model may preserve the CPT symmetry for a single particular choice of v^{μ}. The solution of the wave equations shows a behavior similar but which deviates from the usual MCS electrodynamics by some correction-terms (dependent on the background field). These solutions also indicate the existence of spatial-anisotropy in the case v^{μ} is purely space-like, which is consistent with the determination of a privileged direction is space, $\overrightarrow{v}$. The reduced model exhibits stability, but the causality can be jeopardized by some modes.

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I. INTRODUCTION

In a common sense, it is generally settled that a "good" Quantum Field Theory (QFT) must respect at least two symmetries: the Lorentz covariance and the CPT invariance. The traditional framework of a local QFT, from which one derives the Standard Model that sets the physics inherent to the fundamental particles, satisfies both these symmetries. In the beginning of 90´s, a new work [1] proposing a correction term to the conventional Maxwell Electrodynamics, that preserves the gauge invariance despite breaking the Lorentz, CPT and parity symmetries, was first analyzed. The correction term, composed by the gauge potential, A_{μ}, and an external background 4-vector, v_{μ}, has a Chern-Simons-like structure, $\epsilon^{\mu\nu\kappa\lambda}v_{\mu}A_{\nu}F_{\kappa\lambda}$, and is responsible by inducing an optical activity of the vacuum - or birefringence - among other effects. In this same work, however, it is shown that astrophysical data do not support the birefringence and impose stringent limits on the value of the constant vector v_{μ}, reducing it to a negligible correction term. Similar conclusions, also based on astrophysical observations, were also confirmed by Goldhaber & Timble [3]. Some time later, Colladay and Kostelecky [2] adopted a quantum field theoretical framework to address the issue of CPT- and Lorentz-breakdown as a spontaneous violation. In this sense, they constructed an extension to the minimal Standard Model, which maintains unaffected the $SU(3) \times SU(2) \times U(1)$ gauge structure of the usual theory, and incorporates the CPT-violation as an active feature of the effective low-energy broken action. They started from a usual CPT- and Lorentz-invariant action as defining the properties of what would be an underlying theory at the Planck scale [4], which then suffers a spontaneous breaking of both these symmetries. In the broken phase, there rises the effective action, endowed with breakdown of CPT and Lorentz symmetries, but conservation of covariance under the perspective of the observer inertial frame. The Lorentz invariance is spoiled at the level of the particle-system, which can be viewed in terms of the non-invariance of the fields under boost and Lorentz rotations (relative inertial observer-frames). This covariance breakdown is also manifest when analyzing the dispersion relations, extracted from the propagators.

Investigations concerning the unitarity, causality and consistency of a QFT endowed with violation of Lorentz and CPT symmetries (induced by a Chern-Simons term) were carried out by Adam & Klinkhamer [5]. As result, it was verified that the causality and unitarity of this kind of model can be preserved when the fixed (background) 4-vector is space-like, and spoiled whenever it is time-like or null. A consistency analysis of this model, carried out in the additional presence of a scalar sector endowed with spontaneous symmetry breaking (SSB) [7], has confirmed the results obtained in ref. [5], that is: the space-like case is free from unitarity illnesses, which arise in the time- and light-like cases.

The active development of Lorentz- and CPT-violating theories in $D = 1 + 3$ has come across the inquiry about the structure of a similar model in 1+2 dimensions and its possible implications. In order to study a planar theory, endowed with Lorentz- and CPT-violation, one has decided to adopt a dimensional reduction procedure, that is: one starts from the original Chern-Simons-like term, $\epsilon^{\mu\nu\kappa\lambda}v_\mu A_\nu F_{\kappa\lambda}$, promoting its systematic reduction to $D = 1 + 2$, which yields a pure Chern-Simons term and a Lorentz non-invariant mixing term. Our objective, therefore, is to achieve a planar model, whose structure is derived from a known counterpart defined in 1+3 dimensions, and to investigate some of its features, like propagators, equations of motion for field-strengths and potentials, dispersion relations, causality and stability.

More specifically, one preforms the dimensional reduction to 1+2 dimensions of the Abelian gauge invariant model with non-conservation of the Lorentz and CPT symmetries [1], [5] induced by the term $\epsilon^{\mu\nu\kappa\lambda}v_\mu A_\nu F_{\kappa\lambda}$, resulting in a gauge invariant Planar Quantum Electrodynamics (QED_3) composed by a Maxwell-Chern-Simons gauge field (A_μ), by a scalar field (φ), a scalar parameter (s) without dynamics (the Chern-Simons mass), and a fixed 3-vector (v^μ). Besides the MCS sector, this Lagrangian has a massless scalar sector, represented by the field φ, which also works out as the coupling constant in the Chern-Simons-like structure that mixes the gauge field to the 3-vector, v^μ (where one gauge field is replaced by v^μ). This latter term is the responsible by the Lorentz noninvariance. Therefore, the reduced Lagrangian is endowed with three coupled sectors: a MCS sector, a massless Klein-Gordon sector and a mixing Lorentz-violating one. As it is well-known, the MCS sector breaks both parity and time-reversal symmetries, but preserves the Lorentz and CPT ones. The scalar sector preserves all discrete symmetries and Lorentz covariance, whereas the mixing sector, as it will be seen, breaks Lorentz invariance (in relation to the particle-frame), keeps conserved parity and charge-conjugation symmetries, but may break or preserve time-reversal symmetry. This implies that it may occur both conservation (for a purely space-like v^μ) and violation (for v^μ time-like and light-like) of the CPT invariance.

In short, this paper is outlined as follows. In Section II, one accomplishes the dimensional reduction, that leads to the reduced model. Having established the new planar Lagrangian, one then devotes some algebraic effort for the derivation of the propagators of the gauge and scalar fields, which requires the evaluation of a closed algebra composed by eleven projector operators, displayed into Table I. In Section III, one writes the classical equations of motion (the extended Maxwell equations) and wave equations (for the potential A^μ) correspondent to the reduced Lagrangian. On then verifies that these equations have a similar structure to the usual MCS case, supplemented by terms that depend on the background field. Solving these equations, we obtain solutions that differ from the MCS ones also by v^μ-dependent correction-terms. Concerning the search for some spatial-anisotropy effect induced by the external vector, we remark that the purely time-like solutions, as expected, do not exhibit any signal of this anisotropy, whereas in the purely space-like case the solutions appear with clear dependence on the angle relative to the fixed direction determined by the background, $\overrightarrow{v}$. In Section IV, we investigate the stability and the causal structure of the theory. One addresses the causality looking directly at the dispersion relations extracted from the poles of the propagators, which reveal the existence of both causal and non-causal modes. All the modes, nevertheless, present positive definite energy (positivity) relative to any Lorentz frame, which implies stability. In Section V, we present our Concluding Comments.

II. THE DIMENSIONALLY REDUCED MODEL

One starts from the Maxwell Lagrangian[1] in 1+3 dimensions supplemented by a term that couples the dual electromagnetic tensor to a fixed 4-vector, v^μ, as it appears in ref. [1]:

$$\mathcal{L}_{1+3} = \left\{ -\frac{1}{4}F_{\hat{\mu}\hat{\nu}}F^{\hat{\mu}\hat{\nu}} + \frac{1}{2}\epsilon^{\hat{\mu}\hat{\nu}\hat{\kappa}\hat{\lambda}}v_{\hat{\mu}}A_{\hat{\nu}}F_{\hat{\kappa}\hat{\lambda}} + A_{\hat{\nu}}J^{\hat{\nu}} \right\}, \tag{1}$$

with the additional presence of the coupling between the gauge field and the external current, $A_{\hat{\nu}}J^{\hat{\nu}}$. This model (in its free version) is gauge invariant but does not preserve Lorentz and CPT symmetries relative to the particle frame. For the observer system, the Chern-Simons-like term transforms covariantly, once the background also is changed under

[1]Here one has adopted the following metric conventions: $g_{\mu\nu} = (+, -, -, -)$ in $D = 1 + 3$, and $g_{\mu\nu} = (+, -, -)$ in $D = 1 + 2$. The greek letters (with hat) $\hat{\mu}$, runs from 0 to 3, while the pure greek letters, μ, run from 0 to 2.

an observer boost: $v^{\hat{\mu}} \longrightarrow v^{\hat{\mu}'} = \Lambda^{\mu}{}_{\alpha} v^{\alpha}$. In connection with the particle-system, however, when one applies a boost on the particle, the background 4-vector is supposed to remain unaffected, behaving like a set of four independent numbers, which configures the breaking of the covariance. This term also breaks the parity symmetry, but maintain invariance under charge conjugation and time reversal. To study this model in 1+2 dimensions, one performs its dimensional reduction, which consists effectively in adopting the following ansatz over any 4-vector: (i) one keeps unaffected the temporal and also the first two spatial components; (ii) one freezes the third spacial dimension by splitting it from the body of the new 3-vector and requiring that the new quantities (χ), defined in 1+2 dimensions, do not depend on the third spacial dimension: $\partial_3 \chi \longrightarrow 0$. Applying this prescription to the gauge 4-vector, $A^{\hat{\mu}}$, and to the fixed external 4-vector, $v^{\hat{\mu}}$, and to the 4-current, $J^{\hat{\mu}}$, one has:

$$A^{\hat{\mu}} \longrightarrow (A^{\mu};\ \varphi), \tag{2}$$

$$v^{\hat{\mu}} \longrightarrow (v^{\mu};\ s), \tag{3}$$

$$J^{\hat{\mu}} \longrightarrow (J^{\mu};\ J), \tag{4}$$

where: $A^{(3)} = \varphi$, $v^{(3)} = s$, $J^{(3)} = J$ and $\mu = 0, 1, 2$. According to this process, there appear two scalars: the scalar field, φ, that exhibits dynamics, and s, a constant scalar (without dynamics). Carrying out this prescription for eq. (1), one then obtains:

$$\mathcal{L}_{1+2} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - \frac{s}{2} \epsilon_{\mu\nu k} A^\mu \partial^\nu A^k + \varphi \epsilon_{\mu\nu k} v^\mu \partial^\nu A^k - \frac{1}{2\alpha} \left(\partial_\mu A^\mu\right)^2 + A_\mu J^\mu + \varphi J, \tag{5}$$

where the last free term represents the gauge-fixing term, added up after the dimensional reduction. The scalar field, φ, exhibits a typical Klein-Gordon massless dynamics and it also appears as the coupling constant that links the fixed v^μ to the gauge sector of the model, by means of the new term: $\varphi \epsilon_{\mu\nu k} v^\mu \partial^\nu A^k$. In spite of being covariant in form, this kind of term breaks the Lorentz symmetry in the particle-frame, since the 3-vector v^μ is not sensitive to particle Lorentz boost, behaving like a set of three scalars.

The Lagrangian (1), originally proposed by Carroll-Field-Jackiw [1], has the property of breaking parity symmetry, even though conserving time reversal and charge conjugation symmetries, resulting in nonconservation of the CPT symmetry. Simultaneously, the Lorentz invariance is spoiled, since the fixed 4-vector v^μ breaks the rotational and boost invariances. On the other hand, the reduced model, given by eq.(5), does not necessarily jeopardize the CPT conservation, which depends truly on the character of the fixed vector v^μ. As it is known, the parity transformation ($\mathcal{P}$) in 1+2 dimensions is characterized by the inversion of only on of the spatial axis: $x^\mu \xrightarrow{\mathcal{P}} x'^\mu = (x_o, -x, y)$, the same being valid for the 3-potential: $A^\mu \xrightarrow{\mathcal{P}} A'^\mu = (A_0, -A^{(1)}, A^{(2)})$. The time-reversal transformation ($\mathcal{T}$) must keep unchanged the dynamics of the system, so that one must have: $x^\mu \xrightarrow{\mathcal{T}} x'^\mu = (-x_o, x, y)$, $A^\mu \xrightarrow{\mathcal{T}} A'^\mu = (A_0, -A^{(1)}, -A^{(2)})$, while the charge conjugation determines: $x^\mu \xrightarrow{\mathcal{C}} x'^\mu = x^\mu$, $A^\mu \xrightarrow{\mathcal{C}} A'^\mu = -A^\mu$. One knows that the Chern-Simons term breaks both parity and time-reversal symmetries and keeps conserved the charge conjugation, which assures the global CPT invariance. The new term, $\varphi \varepsilon_{\mu\nu k} v^\mu \partial^\nu A^k$, however, will manifest a non-symmetric behaviour before $\mathcal{T}$-transformation: there will occur conservation if one works with a purely space-like external vector ($v^\mu = (0, \overrightarrow{v})$), or breakdown, if v^μ is purely time-like. Under parity and charge conjugation transformations, in turn, this term will evidence non-invariance for any adopted v^μ, thereby one can state that it will occur CPT conservation when v^μ is purely space-like, and CPT violation otherwise. Here, the field φ was considered as having a scalar character under the parity transformation. Yet, if this field behaves like a pseudo-scalar[2], the CPT conversation will be assured for a purely time-like v^μ. For a light-like v^μ, there will always occur time-reversal non-invariance, and consequently, CPT violation.

Neglecting divergence terms, one can write the linearized free action in an explicitly quadratic form, namely:

$$\Sigma_{1+2} = \int d^3x \frac{1}{2} \Big\{ A^\mu [M_{\mu\nu}] A^\nu - \varphi \Box \varphi + \varphi \left[\epsilon_{\mu\alpha\nu} v^\mu \partial^\alpha\right] A^\nu + A^\mu \left[\epsilon_{\nu\alpha\mu} v^\nu \partial^\alpha\right] \varphi \Big\}, \tag{6}$$

which can also appear in the matrix form:

[2] The adoption of a pseudo-scalar field can be justified by looking at the vector character of the potential ($\overrightarrow{A} \xrightarrow{\mathcal{P}} -\overrightarrow{A}$) before the dimensional reduction. If one assumes that the field φ maintains the same behaviour of its ancestral (A_3), one has a pseudo-scalar.

$$\Sigma_{1+2} = \int d^3x \frac{1}{2} \left(\begin{array}{cc} A^\mu & \varphi \end{array} \right) \left[\begin{array}{cc} M_{\mu\nu} & T_\mu \\ -T_\nu & -\Box \end{array} \right] \left(\begin{array}{c} A^\nu \\ \varphi \end{array} \right). \tag{7}$$

The action (7) has as nucleus a square matrix, P, composed by the quadratic operators of the initial action. The mass dimension of the physical parameters and tensors are: $[A^\mu] = [\varphi] = 1/2$, $[v^\mu] = [s] = 1, [T_\mu] = [M_{\mu\nu}] = 2$. Here, some definitions are necessary:

$$M_{\mu\nu} = \Box\theta_{\mu\nu} + s\ S_{\mu\nu} + \frac{\Box}{\alpha}\omega_{\mu\nu}, \qquad T_\mu = S_{\mu\nu}v^\mu, \tag{8}$$

$$S_{\mu\nu} = \varepsilon_{\mu\kappa\nu}\partial^\kappa,\ \theta_{\mu\nu} = \eta_{\mu\nu} - \omega_{\mu\nu}, \quad \omega_{\mu\nu} = \frac{\partial_\mu\partial_\nu}{\Box}, \tag{9}$$

where $\theta_{\mu\nu}$, $\omega_{\mu\nu}, S_{\mu\nu}$ stand respectively for the transverse, longitudinal and Chern-Simons dimensionless projectors, while $M_{\mu\nu}$ is the quadratic operator associated to the MCS sector. The inverse of the square matrix P, given at the action (7), yields the propagators of the gauge and the scalar fields, which are also written in a matrix form, the propagator-matrix (Δ):

$$\Delta = P^{-1} = \frac{-1}{(\Box M_{\mu\nu} - T_\mu T_\nu)} \left[\begin{array}{cc} -\Box & T_\nu \\ -T_\mu & M_{\mu\nu} \end{array} \right], \tag{10}$$

The propagator of the gauge field, Δ_{11}, and of the scalar field, Δ_{22}, are written as:

$$(\Delta_{11})^{\mu\nu} = \left[\Box\theta_{\mu\nu} + s\ S_{\mu\nu} + \frac{\Box}{\alpha}\omega_{\mu\nu} - \frac{1}{\Box}T_\mu T_\nu\right]^{-1}, \tag{11}$$

$$(\Delta_{22}) = -\frac{M_{\mu\nu}}{\Box}\left[\Box\theta_{\mu\nu} + s\ S_{\mu\nu} + \frac{\Box}{\alpha}\omega_{\mu\nu} - \frac{1}{\Box}T_\mu T_\nu\right]^{-1}, \tag{12}$$

$$(\Delta_{12})^{\mu} = -\frac{T_\nu}{\Box}\left[\Box\theta_{\mu\nu} + s\ S_{\mu\nu} + \frac{\Box}{\alpha}\omega_{\mu\nu} - \frac{1}{\Box}T_\mu T_\nu\right]^{-1}, \tag{13}$$

$$(\Delta_{21})^{\nu} = \frac{T_\mu}{\Box}\left[\Box\theta_{\mu\nu} + s\ S_{\mu\nu} + \frac{\Box}{\alpha}\omega_{\mu\nu} - \frac{1}{\Box}T_\mu T_\nu\right]^{-1}, \tag{14}$$

while the terms Δ_{12}, Δ_{21} are related to the mixed propagators $\langle\varphi A_\mu\rangle$, $\langle A_\mu\varphi\rangle$, that indicate a scalar mediator turning into a gauge mediator and vice-versa. Here, for future purposes, it is useful to present the inverse of the tensor $M_{\mu\nu}$, that is, the propagator of the pure MCS Lagrangian:

$$(M_{\mu\nu})^{-1} = \frac{1}{\Box + s^2}\theta^{\mu\nu} - \frac{s}{\Box(\Box + s^2)}S^{\mu\nu} + \frac{\alpha}{\Box}\omega^{\mu\nu}, \tag{15}$$

To perform the inversion of the operator above, one needs to define some new operators, since the ones known so far do not form a closed algebra, as it is shown below:

$$S_{\mu\nu}T^\nu T^\alpha = \Box v_\mu T^\alpha - \lambda T^\alpha \partial_\mu = \Box Q_\mu{}^\alpha - \lambda \Phi^\alpha{}_\mu, \tag{16}$$

$$Q_{\mu\nu}Q^{\alpha\nu} = T^2 v^\alpha v_\mu = T^2 \Lambda^\alpha{}_\mu, \tag{17}$$

$$Q_{\mu\nu}\Phi^{\nu\alpha} = T^2 v_\mu \partial^\alpha = T^2 \Sigma_\mu{}^\alpha, \tag{18}$$

where the new operators are:

$$Q_{\mu\nu} = v_\mu T_\nu, \quad \Lambda_{\mu\nu} = v_\mu v_\nu, \quad \Sigma_{\mu\nu} = v_\mu \partial_\nu, \quad \Phi_{\mu\nu} = T_\mu \partial_\nu, \tag{19}$$

and,

$$\lambda \equiv \Sigma_\mu^{\ \mu} = v_\mu \partial^\mu\ , \quad T^2 = T_\alpha T^\alpha = (v^2\Box - \lambda^2). \tag{20}$$

Their mass dimension are: $[\Lambda_{\mu\nu}] = 2$, $[Q_{\mu\nu}] = 3$, $[\Sigma_{\mu\nu}] = 2$, $[\Phi_{\mu\nu}] = 3$.

Three of these new terms exhibit a non-symmetric structure, which leads to their consideration in pairs, namely: $Q_{\mu\nu}, Q_{\nu\mu}$; $\Sigma_{\mu\nu}, \Sigma_{\nu\mu}$; $\Phi_{\mu\nu}, \Phi_{\nu\mu}$. The inversion of the operator Δ_{11} will be realized following the traditional prescription,

$\left(\Delta_{11}^{-1}\right)_{\mu\nu}\left(\Delta_{11}\right)^{\nu\alpha}=\delta_{\mu}^{\alpha}$, where the operator $\left(\Delta_{11}\right)^{\nu\alpha}$ is composed by all the possible tensor combinations (of rank two) involving $T_\mu, v_\mu, \partial_\alpha$. In such way, the proposed propagator will consist, at a first glance, of eleven terms:

$$\left(\Delta_{11}\right)^{\nu\alpha}=a_1\theta^{\nu\alpha}+a_2\omega^{\nu\alpha}+a_3S^{\nu\alpha}+a_4\Lambda^{\nu\alpha}+a_5T^{\nu}T^{\alpha}+a_6Q^{\nu\alpha}+a_7Q^{\alpha\nu}+a_8\Sigma^{\nu\alpha}+a_9\Sigma^{\alpha\nu}+a_{10}\Phi^{\nu\alpha}+a_{11}\Phi^{\alpha\nu}, \tag{21}$$

which are displayed in Table I, where one observes explicitly the closure of the operator algebra.

	$\theta_{\mu\nu}$	$\omega_{\mu\nu}$	$S_{\mu\nu}$	$\Lambda_{\mu\nu}$	$T_\mu T_\nu$	$Q_{\mu\nu}$	$Q_{\nu\mu}$	$\Sigma_{\mu\nu}$	$\Sigma_{\nu\mu}$	$\Phi_{\mu\nu}$	$\Phi_{\nu\mu}$
$\theta^{\nu\alpha}$	${\theta_\mu}^\alpha$	0	${S_\mu}^\alpha$	${\Lambda_\mu}^\alpha+$ $-\frac{\lambda}{\Box}{\Sigma_\mu}^\alpha$	$T_\mu T^\alpha$	${Q_\mu}^\alpha$	${Q^\alpha}_\mu+$ $-\frac{\lambda}{\Box}{\Phi_\mu}^\alpha$	0	${\Sigma^\alpha}_\mu+$ $-\lambda\Box{\omega^\alpha}_\mu$	0	${\Phi^\alpha}_\mu$
$\omega^{\nu\alpha}$	0	${\omega_\mu}^\alpha$	0	$\frac{\lambda}{\Box}{\Sigma_\mu}^\alpha$	0	0	$\frac{\lambda}{\Box}{\Phi_\mu}^\alpha$	${\Sigma_\mu}^\alpha$	$\lambda{\omega^\alpha}_\mu$	${\Phi^\alpha}_\mu$	0
$S^{\nu\alpha}$	${S_\mu}^\alpha$	0	$-\Box{\theta_\mu}^\alpha$	${Q_\mu}^\alpha$	$\lambda{\Phi_\mu}^\alpha+$ $-\Box Q^\alpha_\mu$	$\lambda{\Sigma_\mu}^\alpha+$ $-{\Lambda_\mu}^\alpha\Box$	$-T_\mu T^\alpha$	0	$\partial_\mu T^\alpha$	0	$\Box({\omega_\mu}^\alpha+$ $-{\Sigma^\alpha}_\mu)$
$\Lambda^{\nu\alpha}$	${\Lambda_\mu}^\alpha+$ $-\frac{\lambda}{\Box}{\Sigma^\alpha}_\mu$	$\frac{\lambda}{\Box}{\Sigma^\alpha}_\mu$	$-{Q^\alpha}_\mu$	$v^2{\Lambda_\mu}^\alpha$	0	0	$v^2{Q^\alpha}_\mu$	$\lambda{\Lambda_\mu}^\alpha$	$v^2{\Sigma^\alpha}_\mu$	$\lambda{Q^\alpha}_\mu$	0
$T^\nu T^\alpha$	$T_\mu T^\alpha$	0	$\Box{Q_\mu}^\alpha+$ $-\lambda{\Phi^\alpha}_\mu$	0	$T^2T_\mu T^\alpha$	$T^2Q^\alpha_\mu$	0	0	0	0	$T^2{Q_\mu}^\alpha$
$Q^{\nu\alpha}$	${Q_\mu}^\alpha+$ $-\frac{\lambda}{\Box}{\Phi^\alpha}_\mu$	$\frac{\lambda}{\Box}{\Phi^\alpha}_\mu$	$-T_\mu T^\alpha$	$v^2{Q_\mu}^\alpha$	0	0	$v^2T_\mu T^\alpha$	$\lambda{Q_\mu}^\alpha$	$v^2\partial_\mu T^\alpha$	$\lambda T_\mu T^\alpha$	0
$Q^{\alpha\nu}$	${Q^\alpha}_\mu$	0	$\Box{\Lambda_\mu}^\alpha+$ $-\lambda{\Sigma^\alpha}_\mu$	0	$T^2{Q^\alpha}_\mu$	$T^2{\Lambda^\alpha}_\mu$	0	0	0	0	$T^2{\Sigma^\alpha}_\mu$
$\Sigma^{\nu\alpha}$	${\Sigma_\mu}^\alpha+$ $-\lambda{\omega_\mu}^\alpha$	$\lambda{\omega_\mu}^\alpha$	$-{\Phi_\mu}^\alpha$	$v^2{\Sigma_\mu}^\alpha$	0	0	$v^2{\Phi_\mu}^\alpha$	$\lambda{\Sigma_\mu}^\alpha$	$v^2{\Lambda_\mu}^\alpha$	$\lambda{\Phi_\mu}^\alpha$	0
$\Sigma^{\alpha\nu}$	0	${\Sigma^\alpha}_\mu$	0	$\lambda{\Lambda_\mu}^\alpha$	0	0	$\lambda{Q^\alpha}_\mu$	$\Box{\Lambda_\mu}^\alpha$	$v^2{\Lambda_\mu}^\alpha$	$\Box{Q_\mu}^\alpha$	0
$\Phi^{\nu\alpha}$	${\Phi_\mu}^\alpha$	0	$\Box({\Sigma_\mu}^\alpha+$ $-\lambda{\omega_\mu}^\alpha)$	0	$T^2{\Phi_\mu}^\alpha$	$T^2{\Sigma_\mu}^\alpha$	0	0	0	0	$\Box T^2\omega^\alpha_\mu$
$\Phi^{\alpha\nu}$	0	${\Phi^\alpha}_\mu$	0	$\lambda{\Phi^\alpha}_\mu$	0	0	$\lambda T_\mu T^\alpha$	$\Box{\Phi^\alpha}_\mu$	$\lambda{\Phi^\alpha}_\mu$	$\Box T_\mu T^\alpha$	0

Table I: Multiplicative operator algebra fulfilled by θ, ω, S, Λ,TT, Q, Σ,and Φ. The products are supposed to be in the ordering "row times column".

Using the data contained in Table I, one finds out that the gauge-field propagator assumes the form:

$$\begin{aligned}\left(\Delta_{11}\right)^{\mu\nu} &= \frac{1}{\Box+s^2}\theta^{\mu\nu}+\frac{\alpha(\Box+s^2)\boxtimes-\lambda^2s^2}{\Box(\Box+s^2)\boxtimes}\omega^{\mu\nu}-\frac{s}{\Box(\Box+s^2)}S^{\mu\nu}-\frac{s^2}{(\Box+s^2)\boxtimes}\Lambda^{\mu\nu}+\frac{1}{(\Box+s^2)\boxtimes}T^\mu T^\nu \\ &\quad -\frac{s}{(\Box+s^2)\boxtimes}Q^{\mu\nu}+\frac{s}{(\Box+s^2)\boxtimes}Q^{\nu\mu}+\frac{\lambda s^2}{\Box(\Box+s^2)\boxtimes}\Sigma^{\mu\nu}+\frac{\lambda s^2}{\Box(\Box+s^2)\boxtimes}\Sigma^{\nu\mu}-\frac{s\lambda}{\Box(\Box+s^2)\boxtimes}\Phi^{\mu\nu} \\ &\quad +\frac{s\lambda}{\Box(\Box+s^2)\boxtimes}\Phi^{\nu\mu},\end{aligned}$$

where: $\boxtimes=(\Box^2+s^2\Box-T^2)$.

By the same procedure, one evaluates the mixed propagator, $\left(\Delta_{12}\right)^{\alpha}=-\frac{T_\nu}{\Box}\left(\Delta_{11}\right)^{\nu\alpha}$, which can be written in the following form:

$$\left(\Delta_{12}\right)^{\nu}=-\frac{1}{\boxtimes}\left[T^\nu+sv^\nu-\frac{s\lambda}{\Box}\partial^\nu\right], \tag{22}$$

whereas the propagator $\left(\Delta_{21}\right)^{\nu}$, in turn, results exactly equal to $\left(\Delta_{12}\right)^{\nu}$. In order to compute the propagator of the scalar field,

$$\left(\Delta_{22}\right)=-\frac{1}{\Box}\left[1-\frac{1}{\Box}T_\mu\left(M_{\mu\nu}\right)^{-1}T_\nu\right]^{-1}, \tag{23}$$

one makes use of the inverse of the tensor $M_{\mu\nu}$, given by eq. (15), so that: $T_\mu(M^{-1})^{\mu\nu}T_\nu=\left(\Box+s^2\right)^{-1}T^2$. In such a way, a compact scalar propagator arises:

$$(\Delta_{22}) = -\frac{\Box + s^2}{\boxtimes} \tag{24}$$

In momentum-space, the photon propagator takes the final expression:

$$\begin{aligned}\langle A^{\mu}(k) A^{\nu}(k)\rangle = i\Big\{ & -\frac{1}{k^2-s^2}\theta^{\mu\nu} - \frac{\alpha(k^2-s^2)\boxtimes(k) + s^2(v_\alpha k^\alpha)^2}{k^2(k^2-s^2)\boxtimes(k)}\omega^{\mu\nu} - \frac{s}{k^2(k^2-s^2)}S^{\mu\nu} + \frac{s^2}{(k^2-s^2)\boxtimes(k)}\Lambda^{\mu\nu} \\ & -\frac{1}{(k^2-s^2)\boxtimes(k)}T^\mu T^\nu + \frac{s}{(k^2-s^2)\boxtimes(k)}Q^{\mu\nu} - \frac{s}{(k^2-s^2)\boxtimes(k)}Q^{\nu\mu} + \frac{is^2(v_\alpha k^\alpha)}{k^2(k^2-s^2)\boxtimes(k)}\Sigma^{\mu\nu} \\ & + \frac{is^2(v_\alpha k^\alpha)}{k^2(k^2-s^2)\boxtimes(k)}\Sigma^{\nu\mu} - \frac{is(v_\alpha k^\alpha)}{k^2(k^2-s^2)\boxtimes(k)}\Phi^{\mu\nu} + \frac{is(v_\alpha k^\alpha)}{k^2(k^2-s^2)\boxtimes(k)}\Phi^{\nu\mu}\Big\},\end{aligned} \tag{25}$$

while the scalar and the mixed propagators read as:

$$\langle\varphi\varphi\rangle = \frac{i}{\boxtimes(k)}\left[k^2 - s^2\right], \tag{26}$$

$$\langle\varphi A^\alpha(k)\rangle = -\frac{i}{\boxtimes(k)}\left[T^\alpha + sv^\alpha - \frac{s(v_\mu k^\mu)}{k^2}k^\alpha\right], \tag{27}$$

where: $\boxtimes(k) = \left[k^4 - \left(s^2 - v^2\right)k^2 - (v_\mu k^\mu)^2\right]$. By the above expressions, one notes that the factor $\boxtimes$ is present on the denominator of all propagators, in such a way the scalar and the gauge field will share the pole structure, and consequently, the physical excitations associated with the poles of $\boxtimes(k)$. This common dependence on $1/\boxtimes$ also amounts to similarities on the causal structure of the scalar and gauge sectors of this model, as it will de discussed in Section IV.

III. CLASSICAL WAVE EQUATIONS AND SOLUTIONS

Let us now consider the reduced model, given by Lagrangian (5), without the gauge-fixing term:

$$\mathcal{L}_{1+2} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - \frac{s}{2}\epsilon_{\mu\nu k}A^\mu\partial^\nu A^k + \varphi\varepsilon_{\mu\nu k}v^\mu\partial^\nu A^k + A_\mu J^\mu + \varphi J, \tag{28}$$

where one observes the Chern-Simons term (having s as topological mass) and the Lorentz-violating term that couples the fixed 3-vector v^μ to the gauge vector A^μ. Associated to this Lagrangian there are two Euler-Lagrangian motion equations:

$$\partial_\nu F^{\mu\nu} = -\frac{s}{2}\varepsilon^{\mu\nu\rho}\partial_\nu A_\rho - \varepsilon^{\mu\nu\rho}v_\nu\partial_\rho\varphi - J^\mu, \tag{29}$$

$$\Box\varphi = \epsilon_{\mu\nu k}v^\mu\partial^\nu A^k - J. \tag{30}$$

The modified Maxwell equations associated with this Lagrangian read as below:

$$\overrightarrow{\nabla} \times \overrightarrow{E} + \partial_t B = 0, \tag{31}$$

$$\partial_t\overrightarrow{E} - \nabla^* B = -\overrightarrow{j} + s\overrightarrow{E}^* + \left(\overrightarrow{v}^*\partial_t\varphi + v_0\overrightarrow{\nabla}^*\varphi\right), \tag{32}$$

$$\overrightarrow{\nabla}.\overrightarrow{E} + sB = \rho - \overrightarrow{v} \times \overrightarrow{\nabla}\varphi, \tag{33}$$

$$\Box\varphi - \overrightarrow{v} \times \overrightarrow{E} = -v_0\overrightarrow{\nabla} \times \overrightarrow{A} - J, \tag{34}$$

where the first equation stems from the Bianchi identity[3] ($\partial_\mu F^{\mu *} = 0$), while the two inhomogeneous ones come from the motion equation (29), and the last one is derived from eq. (30). Explicitly, one notes that Eq.(30) can

[3] In $D = 1+2$ the dual tensor, defined as $F^{\mu *} = \frac{1}{2}\epsilon^{\mu\nu\alpha}F_{\nu\alpha}$, is a 3-vector given by: $F^{\mu *} = (B, -\overrightarrow{E}^*)$. Here one adoptes the following convection: $\epsilon_{012} = \epsilon^{012} = \epsilon_{12} = \epsilon^{12} = 1$. The symbol $(^*)$, in a general way, also designates the dual of a 2-vector: $\left(E^i\right)^* = \epsilon_{ij}E^j \longrightarrow \overrightarrow{E}^* = (E_y, -E_x)$.

be written as two simpler equations whether the vector v^{μ} is purely space- or time-like: $\Box\varphi = \overrightarrow{v} \times \overrightarrow{E} - J$, for $v^{\mu} = (0, \overrightarrow{v})$; $\Box\varphi = -v_0 \overrightarrow{\nabla} \times \overrightarrow{A} - J$, for $v^{\mu} = (v_0, \overrightarrow{0})$. Applying the differential operator, ∂_{μ}, on the eq. (29), there results the following equation for the gauge current: $\partial_{\mu}J^{\mu} = -\varepsilon^{\mu\nu\rho}\partial_{\mu}v_{\nu}\partial_{\rho}\varphi$, which reduces to the conventional current-conservation law, $\partial_{\mu}J^{\mu} = 0$, when v^{μ} is constant or has a null rotational ($\varepsilon^{\mu\nu\rho}\partial_{\mu}v_{\nu} = 0$). These conditions correspond exactly to the ones that lead to a gauge invariant theory [1].

Manipulating the Maxwell equations, one notes that the fields B, $\overrightarrow{E}$, satisfy inhomogeneous wave equations:

$$(\Box + s^2)B = s\rho + \overrightarrow{\nabla} \times \overrightarrow{j} - s\overrightarrow{v} \times \nabla\varphi - \partial_t\left(\nabla\varphi\right) \times \overrightarrow{v}^* + v_0\nabla^2\varphi, \tag{35}$$

$$(\Box + s^2)\overrightarrow{E} = -\overrightarrow{\nabla}\rho - \partial_t\overrightarrow{j} - s\overrightarrow{j}^* - s\overrightarrow{v}\left(\partial_t\varphi\right) - sv_o\overrightarrow{\nabla}\varphi + \overrightarrow{v}^*\partial_t^2\varphi + v_0\overrightarrow{\nabla}^*\left(\partial_t\varphi\right) + \overrightarrow{\nabla}(\overrightarrow{v} \times \overrightarrow{\nabla\varphi}), \tag{36}$$

which, in the stationary regime, are reduced to:

$$(\nabla^2 - s^2)B = -s\rho - \overrightarrow{\nabla} \times \overrightarrow{j} + s\overrightarrow{v} \times \nabla\varphi - v_0\nabla^2\varphi, \tag{37}$$

$$(\nabla^2 - s^2)\overrightarrow{E} = s\overrightarrow{j}^* + \overrightarrow{\nabla}\rho + sv_o\overrightarrow{\nabla}\varphi - \overrightarrow{\nabla}(\overrightarrow{v} \times \overrightarrow{\nabla\varphi}). \tag{38}$$

Similarly to the behaviour of the classical MCS-potential, here the potential components $(A_0, \overrightarrow{A})$ obey fourth-order wave equations:

$$\Box(\Box + s^2)A_0 = \Box\rho - \Box(\overrightarrow{v} \times \overrightarrow{\nabla\varphi}) - s\overrightarrow{\nabla} \times \overrightarrow{j} + s\left(\partial_t\overrightarrow{\nabla\varphi}\right) \times \overrightarrow{v} - sv_o\nabla^2\varphi, \tag{39}$$

$$\Box(\Box + s^2)\overrightarrow{A} = s\partial_t\overrightarrow{j}^* + s\overrightarrow{\nabla}^*\rho + s\overrightarrow{v}\left(\partial_t^2\varphi\right) + sv_o\overrightarrow{\nabla}\left(\partial_t\varphi\right) - s\left(\overrightarrow{\nabla}(\overrightarrow{v} \times \overrightarrow{\nabla\varphi})\right)^* + \Box(\overrightarrow{j} - \overrightarrow{v}\partial_t\varphi - v_0\overrightarrow{\nabla}^*\varphi), \tag{40}$$

which are endowed with an inhomogeneous sector much more complex due to the presence of the terms $\overrightarrow{v}$ and φ in the Lagrangian (28). It is instructive to remark that wave equations (35, 36, 39, 40) reduce to their classical MCS usual form [10], [11] in the limit one takes $v^{\mu} \longrightarrow 0$ and $\varphi \longrightarrow 0$, namely:

$$(\Box + s^2)B = s\rho + \overrightarrow{\nabla} \times \overrightarrow{j}; \quad (\Box + s^2)\overrightarrow{E} = -\overrightarrow{\nabla}\rho - \partial_t\overrightarrow{j} - s\overrightarrow{j}^*; \tag{41}$$

$$\Box(\Box + s^2)A_0 = \Box\rho - s\overrightarrow{\nabla} \times \overrightarrow{j}; \quad \Box(\Box + s^2)\overrightarrow{A} = s\partial_t\overrightarrow{j}^* + s\overrightarrow{\nabla}^*\rho + \Box\overrightarrow{j}. \tag{42}$$

The above wave equations present the following solutions [10] (for a point-like charge distribution and null current):

$$B(r) = (e/2\pi)\,K_0(sr); \quad \overrightarrow{E} = (e/2\pi)\,sK_1(sr)\,\hat{r}; \tag{43}$$

$$A_0(r) = (e/2\pi)\,K_0(sr); \quad \overrightarrow{A}(r) = (e/2\pi)\left[1/r - srK_1(sr)\right]\hat{r}^*. \tag{44}$$

Up to now, eq. (30) was not still used in the derivation of the wave equations for the fields and potentials. It will be appropriately considered in the subsequent solutions.

A. Solution for the scalar potential and the strength fields in the static limit

The wave equation that rules the dynamics of the scalar potential, A_0, is already known. Eq. (39), however, is not entirely written in terms of A_0, since the scalar field φ is not a constant variable and exhibits its own dynamics described by eq. (34), which now must be taken into account to provide the correct solution of the wave equation. Eq. (39) will present two different solutions depending on the character of the fixed vector v^{μ}.

1. Case 1: *The external vector is purely time-like:* $v^{\mu} = (v_0, 0)$

Supposing the system reaches a stationary regime, eq. (39) is reduced to

$$\nabla^2(\nabla^2 - s^2)A_0 = -\nabla^2\rho - s\overrightarrow{\nabla} \times \overrightarrow{j} - sv_o\nabla^2\varphi + \nabla^2\left(\overrightarrow{v} \times \overrightarrow{\nabla}\varphi\right). \tag{45}$$

In this case, the φ-field satisfies the equation: $\nabla^2\varphi = v_0B + J$. The use of eq. (31) changes the eq. (45) to the form:

$$\nabla^2(\nabla^2 - s^2 + v_0^2)A_0 = -\nabla^2\rho - s\overrightarrow{\nabla} \times \overrightarrow{j} - v_0^2\rho - J. \tag{46}$$

Starting from a point-like charge-density distribution, $\rho(r) = e\delta(r)$, taking a null current-density, $\overrightarrow{j} = 0, J = 0$, and proposing a Fourier-transform expression for the scalar potential, $A_0(r) = \frac{1}{(2\pi)^2}\int d^2\overrightarrow{k}\, e^{i\overrightarrow{k}.\overrightarrow{r}}\widetilde{A}_0(k)$, it follows as solution,

$$A_0(r) = \frac{e}{(2\pi)w^2}\left[s^2K_0(wr) + v_0^2\ln r\right], \tag{47}$$

where: $w^2 = s^2 - v_0^2$. If $s^2 > v_0^2$, this potential is always repulsive. Moreover, it is trivial to see that in the limit $v_0 \longrightarrow 0$, one recovers the scalar potential associated with the MCS-Electrodynamics, given by eq. (44). It is then clear that the term with dependence on $\ln r$ is then a contribution stemming from the background field. The electric field, derived from eq. (47), is read as,

$$\overrightarrow{E}(r) = \frac{e}{(2\pi)}\left[\frac{s^2}{w}K_1(wr) - \left(\frac{v_0^2}{w^2}\right)\frac{1}{r}\right]\hat{r}, \tag{48}$$

which compared with the MCS correspondent, that of eq. (43), possesses the additional presence of the $1/r$-term, which certainly arises as the contribution of the background, similarly as it occurs at eq. (47). In the limit of short distance $(r \ll 1)$, the scalar potential (47) and the electric field are reduced to the form:

$$A_0(r) = -\frac{e}{(2\pi)}\left[\ln r + \frac{s^2}{w^2}\ln w\right]; \qquad \overrightarrow{E}(r) = \left(\frac{e}{2\pi}\right)\frac{1}{r}\hat{r}; \tag{49}$$

which reveals the repulsive character of expression (47) and a radial $1/r$ electric field near the origin.

In the absence of currents, the magnetic field is ruled by eq. (37), which reads simply as: $(\nabla^2 - s^2 + v_0^2)B = -s\rho$. This differential equation is fulfilled by a very simple solution:

$$B(r) = \left(\frac{es}{2\pi}\right)K_0(wr). \tag{50}$$

In comparing this magnetic field with that of eq. (43), one does not observe any additional term. In this case, the influence of the background seems to be totally absorbed into the decay factor, w. Finally, one can remark that the results here obtained do not exhibit any signal of spatial anisotropy, which is consistent with the adoption of a null vector $\overrightarrow{v}$, since this vector is the element responsible by the choice of a privileged direction in space. The anisotropy, therefore, must be manifest when v^μ is space-like.

2. *Case 2: The external vector is purely space-like:* $v^\mu = (0, v)$

In this case, the equation fulfilled by the scalar field, $\nabla^2\varphi = -\overrightarrow{v}\times\overrightarrow{E}$, can be read in term of the scalar potential: $\nabla^2\varphi = -\overrightarrow{v}\times\overrightarrow{\nabla}A_0 + J = (\overrightarrow{v}.\overrightarrow{\nabla}^*)A_0 + J$. Taking into account this relation, eq. (39) in its stationary regime is reformulated as:

$$\left[\nabla^2(\nabla^2 - s^2) - (\overrightarrow{v}.\overrightarrow{\nabla}^*)(\overrightarrow{v}.\overrightarrow{\nabla}^*)\right]A_0 = -\nabla^2\rho - s\overrightarrow{\nabla}\times\overrightarrow{j} + (\overrightarrow{v}.\overrightarrow{\nabla}^*)J, \tag{51}$$

where it was used the relation: $\nabla^2(\overrightarrow{v}\times\overrightarrow{\nabla\varphi}) = (\overrightarrow{v}.\overrightarrow{\nabla}^*)\nabla^2\varphi = (\overrightarrow{v}.\overrightarrow{\nabla}^*)(\overrightarrow{v}.\overrightarrow{\nabla}^*)A_0$, since $\overrightarrow{v} = cte$.

Starting from a point-like charge density distribution, $\rho(r) = e\delta(r)$, $\overrightarrow{j} = J = 0$, and proposing again the same kind of Fourier-transform expression for the scalar potential, one obtains:

$$A_0(r) = -\frac{e}{(2\pi)^2}\int_0^\infty kdk\int_0^{2\pi}d\varphi\frac{e^{ikr\cos\varphi}}{\left[(\overrightarrow{k}^2 + s^2) + \overrightarrow{v}^2\sin^2\alpha\right]}, \tag{52}$$

where α is the angle defined by: $\overrightarrow{v}.\overrightarrow{k} = vk\cos\alpha$. An exact result was not found for this full integral, but an approximation can be accomplished in order to solve it algebraically. Here, there is an external vector, $\overrightarrow{v}$, that fixes a direction in space and the coordinate position, $\overrightarrow{r}$, where one measures the fields. One then considers that the angle between $\overrightarrow{v}$ and $\overrightarrow{r}$ is given by: $\overrightarrow{v}.\overrightarrow{r} = vr\cos\beta$, where $\beta = cte$. Considering this information and working in limit in which $s^2 \gg v^2$, the integration becomes feasible, so that one attains (at first order on v^2/s^2):

$$A_0(r) = \frac{e}{(2\pi)}\left[K_0(sr) - \frac{(1-\cos^2\beta)}{2s}v^2rK_1(sr) + \frac{v^2}{2s^2}\left(1-\cos^2\beta\right)K_2(sr)\right]. \tag{53}$$

In this expression, one notes a clear dependence of the potential on the angle β, which is a unequivocal sign of anisotropy determined by the ubiquity of background vector on the system. Near the origin, the K_2-function dominates over the other terms, so that the short-distance potential behaves effectively as:

$$A_0(r) = \frac{e}{(2\pi)}\left[\left(1-\cos^2\beta\right)\frac{v^2}{s^2}\frac{1}{r^2}\right], \tag{54}$$

which shows that the potential is always repulsive at origin. In spite of this fact, the expression (53) may exhibit an attractive well region, at larger r-values, depending on the value of the s parameter. This fact brings into light the possibility of occurrence of pair-condensation concerning two particles interacting by means this gauge field. This issue should be more properly investigated in the context of the a low-energy two-particle scattering, whose amplitude can be converted into the interaction potential by a Fourier transform.

Looking at the expression (40) for the vector potential, one observes the presence of the term $\overrightarrow{\nabla}(\overrightarrow{v}\times\overrightarrow{\nabla\varphi})$, which can not be written as a term depending directly on $\overrightarrow{A}$. This fact seems to prevent a solution for $\overrightarrow{A}$ starting from the static version of this differential equation, which also seems to be an impossibility for determining a solution for the magnetic field. However, one must be indeed interested in the magnetic field, and a simpler solution for it can arise from the eq. (32), which in the static regime is simplified to the form: $\nabla B = -s\overrightarrow{E} - v_0\overrightarrow{\nabla}\varphi$. For a pure space-like v^μ this last equation reduces to: $\nabla B = -s\overrightarrow{E} = s\nabla A_0$, an equation that links the magnetic field and the scalar potential: $B = sA_0 + cte$. Based on eq. (53), we achieve the following expression for the fields:

$$\overrightarrow{E}(r) = \frac{e}{(2\pi)}\left[sK_1(sr) + \left(1-\cos^2\beta\right)\frac{v^2}{2}\left[r-\frac{2}{s^2r}\right]K_0(sr) + (1-2\cos^2\beta)\frac{v^2}{2s}\left[1-\frac{4}{s^2r^2}\right]K_1(sr)\right]\hat{r}, \tag{55}$$

$$B(r) = \frac{e}{(2\pi)}\left[sK_0(sr) - \left(1-\cos^2\beta\right)\frac{v^2}{2}rK_1(sr) + \frac{v^2}{2s}\left(1-\cos^2\beta\right)K_2(sr)\right]. \tag{56}$$

Here, the effect of the background vector, $\overrightarrow{v}$, appears more clearly on the field solutions. As compared to the MCS fields (B and $\overrightarrow{E}$), there arise supplementary terms, proportional to $\cos^2\beta$, responsible by the spatial anisotropy.

In order to present, in a naive way, an expression for the density of energy associated with this theory, we look directly at the Maxwell equations. First, we define the field strengths corresponding to the potential φ: $Z = \partial_t\varphi$, $\overrightarrow{T} = \nabla\varphi$, and rewrite the Maxwell equations (31-32) in terms of Z and $\overrightarrow{T}$. Thereafter, we manipulate these equations in such a way to obtain a temporal derivative of a sum of squared fields, that is: $(B^2 + \overrightarrow{E}^2 + Z^2 + \overrightarrow{T}^*.\overrightarrow{T})$. Following this procedure, one achieves an expression analogous to the Poynting Theorem (relating the density of energy and the Poynting vector):

$$\partial_t U + \overrightarrow{\nabla}.\overrightarrow{S} = \overrightarrow{E}.(\overrightarrow{T}^* - \overrightarrow{j}) + Z\frac{v_0}{s}\overrightarrow{v}\times\overrightarrow{T} - Z\frac{v_0}{s}\rho + (\overrightarrow{T}.\overrightarrow{\nabla})Z, \tag{57}$$

where $U = \frac{1}{2}\left[B^2 + \overrightarrow{E}^2 + Z^2 + \overrightarrow{T}^*.\overrightarrow{T}\right]$ and $\overrightarrow{S} = \left[B\overrightarrow{E}^* - Z\overrightarrow{T} - \frac{v_0}{s}Z\overrightarrow{E}\right]$ are respectively the energy density and the Poynting vector. The density of energy here derived is positive definite, which does not anticipate any stability problem. In the next section, one deals again with the stability of this theory, but now under the perspective of the dispersion relations. These results confirm the stability of the theory.

IV. DISPERSION RELATIONS, STABILITY AND CAUSALITY ANALYSIS

Some references in literature [5], [6], [8] have dealt with the issue of stability, causality and unitarity concerning to Lorentz- and CPT-violating theories. The causality is usually addressed as a quantum feature that requires the commutation between observables separated by a space-like interval, which one calls microcausality in field theory [9]. In this section, however, one analyzes causality under a classical tree-level perspective, in which it is related to the positivity of a usual Lorentz invariant, k^2. The starting-point of all investigation is the propagator, whose poles are associated to dispersion relations (DR) that provide informations about the stability and causality of the model. The causality analysis is then related to the sign of the propagator poles, given in terms of k^2, in such a way one must have $k^2 \geq 0$ in order to preserve it (circumventing the existence of tachyons). In the second quantization framework, stability is related to the energy positivity of the Fock states for any momentum. Here, stability is directly associated with the energy positivity of each mode read off from the DR.

The field propagators, given by eqs. (25, 27, 26), present three families of poles at k^2:

$$k^2 = 0; \quad k^2 - s^2 = 0; \quad k^4 - (s^2 - v^2)k^2 - (v.k)^2 = 0, \tag{58}$$

from which one straightforward infers the DR derived from the Lagrangian (5), namely:

$$k_{0(1)}^2 = \overrightarrow{k}^2; \quad k_{0(2)}^2 = \overrightarrow{k}^2 + s^2; \quad k_{0(3)}^2 = \overrightarrow{k}^2 + \frac{1}{2}\left[\left(s^2 - v^2\right) \pm \sqrt{\left(s^2 - v^2\right)^2 + 4\left(v.k\right)^2}\right]. \tag{59}$$

The first dispersion relation, $k_0 = \pm|\overrightarrow{k}|$, stands for a massless photon mode, which carries no degree of freedom, since the Lagrangian (5) involves a massive photon. The second DR represents the Chern-Simons massive mode, $k_0 = \pm\sqrt{s^2 + |\overrightarrow{k}|^2}$, which propagates only one degree of freedom (in the Maxwell-Chern-Simons electrodynamics the scalar magnetic field encloses all information of the electromagnetic field, which justifies the existence of a single degree of freedom). These first two poles apparently respect the causality condition, since $k^2 \geq 0$ for them. Once the causality is set up, the stability comes up as a direct consequence.

Concerning the third DR, corresponding to the roots of $\boxtimes(k)$, it may provide both massless and massive modes for some specific $\overrightarrow{k}$-values, but in general, the mode is massive. By remembering that $\overrightarrow{k}$ is the transfer momentum, whose values are generally integrated from zero to infinity, one concludes it does not make much sense to fix any value for $\overrightarrow{k}$ in order to obtain a particular dispersion relation. Remarking that the term $\boxtimes(k)$ is ubiquitous in the denominator of all propagators, as it is explicit in eqs. (25),(26), (27), one concludes the causal structure entailed to the poles of $1/\boxtimes$ will be common to these three propagators. Specifically, for a purely space-like 3-vector, $v^\mu = (0, \overrightarrow{v})$, this DR is written as,

$$k_{0\pm}^2 = \overrightarrow{k}^2 + \frac{1}{2}\left[\left(s^2 + \overrightarrow{v}^2\right) \pm \sqrt{\left(s^2 + \overrightarrow{v}^2\right)^2 + 4\left(\overrightarrow{v}.\overrightarrow{k}\right)^2}\right]. \tag{60}$$

A simple analysis of this expression indicates that both k_{0+}^2 and k_{0-}^2 are positive-energy modes for any $\overrightarrow{k}$-value (and for any Lorentz observer), which assures the stability of these modes. This fact may suggest that the causal structure of the space-like sector of this model remains preserved, as it was observed by Adam & Klinkhamer [5] in the context of the 4-dimensional version of this theory, that is endowed with a dispersion relation very similar to eq. (60) (this conclusion was also supported by the attainment of a group velocity, associated to this mode, smaller than 1). Concerning the pole analysis, although, we have $k_+^2 > 0$ for arbitrary $\overrightarrow{k}$ and $k_-^2 < 0$ (unless $\overrightarrow{k} \perp \overrightarrow{v}$ or $\overrightarrow{k} = 0$, which implies $k_-^2 = 0$). So, while the mode k_+^2 preserves the causality and stability, the mode k_-^2, in spite of assuring stability, will be in general non-causal, preserving causality only when $\overrightarrow{k} \perp \overrightarrow{v}$ or $\overrightarrow{k} = 0$.

In the case of a purely time-like 3-vector, $v^\mu = (v_0, \overrightarrow{0})$, the DR assumes the form:

$$k_{0\pm}^2 = \frac{1}{2}\left[\left(s^2 + 2\overrightarrow{k}^2\right) \pm \sqrt{s^4 + 4v_o^2\overrightarrow{k}^2}\right], \tag{61}$$

where one observes a similar behaviour: the mode k_{0+}^2 will exhibit stability and causality, while the mode k_{0-}^2 will present energy positivity for arbitrary $\overrightarrow{k}$-value, only if the condition $\left(s^2 - v_0^2 > 0\right)$ is fulfilled. This latter mode is non-causal for any $\overrightarrow{k} \neq 0$. Assuming the coefficients for Lorentz violation are small near the Chern-Simons mass $\left(s^2 \gg v_0^2, |\overrightarrow{v}|^2\right)$, we obtain an entirely causal theory (at least at zero order in v^2/s^2). This is consistent with some results [8] concerning some quantum theories containing Lorentz-violating terms, which evidence the preservation of causality when the breaking factors are small.

Hence, the modes $k_{0\pm}^2$ exhibit positive energy both in space- and time-like cases, which also implies these two modes can be written as an expansion in terms of positive and negative frequency terms. This separation allows the definition of particles and antiparticles states, a necessary condition for the quantization of this theory. Nevertheless, the existence of non-causal modes, both in time- and space-like case, can be seen already at classical level, as a prediction on the impossibility to realize a consistent quantization of this model. The existence of quantization illness is an issue better addressed by investigating the unitarity of the model, matter to be discussed in a forthcoming version of this paper.

In a Lorentz covariant framework, k^2 is a Lorentz scalar, which assures a unique value for all Lorentz frames. In such a way, if k^2 represents a causal mode for one observer, so it will be for all ones. The fact that k^2 has not a positive definite value in an arbitrary Lorentz frame is a unequivocal indicative of the Lorentz covariance breakdown.

V. CONCLUDING COMMENTS

We have accomplished the dimensional reduction to 1+2 dimensions of a gauge invariant, Lorentz and CPT-violating model, defined by the Carroll-Field-Jackiw term, $\epsilon^{\mu\nu\kappa\lambda}v_\mu A_\nu F_{\kappa\lambda}$. One then obtains a Maxwell-Chern-Simons planar

Lagrangian in the presence of a Lorentz breaking term and a massless scalar field. Concerning this reduced model, the CPT symmetry is conserved for a purely space-like v^{μ}, and spoiled otherwise. The propagators of this model are evaluated and exhibit a common causal structure (bound to the dependence on $1/\boxtimes$). One writes the extended Maxwell equations, which in turn determine the wave equations for the field-strengths and potentials. In the purely time-like case, such wave equations afford solutions for the fields B and $\overrightarrow{E}$ that differ from the MCS counterparts just by small correction-terms dependent on v^{μ}. In the pure space-like, the background field appears more explicitly in the solution at the form of spatial anisotropy, a consequence of the selection of a privileged space-time direction by v^{μ}. The poles of the propagators are also used as starting point for the analysis of causality and stability. Concerning the dispersion relations, one verifies that the modes have positive definite energy, which ensures stability. The causality is assured for all modes of the theory, except for k_{-}^{2} (both in space- and time-like case). The investigation of the unitarity of this model is a relevant issue and essential condition to make feasible a consistent quantization of this specific theory. Once the unitarity is ensured, this model may become a useful and interesting tool to analyze planar systems, in the realm of Condensed-Matter, endowed with anisotropy.

A new version of this work [12] may address the dimensional reduction of a gauge-Higgs model [7] in the presence the Carroll-Field-Jackiw term. In this case, the reduced model will be composed by two scalar fields (one stemming from the dimensional reduction and other being the Higgs scalar), by a Maxwell-Chern-Simons-Proca gauge field, and by the Lorentz-violating mixing term. The introduction of the Higgs sector may shed light on new interesting issues concerning planar systems, like the investigation of vortex-like configurations in the framework of a Lorentz-breaking model.

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